Written Exam for the M.Sc. in Economics Summer 2016

# ADVANCED MACROECONOMETRICS

Final Exam

June 10, 10:00 – June 12, 10:00

PLEASE NOTE that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish. If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The paper must be uploaded as one PDF document (including the standard cover and the appendices). The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Absalon.

FOCUS ON EXAM CHEATING: In case of presumed exam cheating, which is observed by either the examination registration of the respective study programmes, the invigilation or the course lecturer, the Head of Studies will make a preliminary inquiry into the matter, requesting a statement from the course lecturer and possibly the invigilation, too. Furthermore, the Head of Studies will interview the student. If the Head of Studies finds that there are reasonable grounds to suspect exam cheating, the issue will be reported to the Rector. In the course of the study and during examinations, the student is expected to conform to the rules and regulations governing academic integrity. Academic dishonesty includes falsification, plagiarism, failure to disclose information, and any other kind of misrepresentation of the student's own performance and results or assisting another student herewith. For example failure to indicate sources in written assignments is regarded as failure to disclose information. Attempts to cheat at examinations are dealt with in the same manner as exam cheating which has been carried through. In case of exam cheating, the following sanctions may be imposed by the Rector:

1. A warning
2. Expulsion from the examination
3. Suspension from the University for at limited period or permanent expulsion.

The Faculty of Social Sciences
The Study and Examination Office
October 2006

# PRACTICAL INFORMATION

Note the following formal requirements:

- This is an *individual* examination. You are not allowed to cooperate with other students or other people, see the *focus on exam cheating* above.
- The assignment consists of Sections 1-7 with 21 questions to be answered. *Please answer all questions.*
- The exam paper should not exceed 20 pages. A maximum of 10 pages of supporting material (graphs, estimation output, etc.) can accompany the paper.
- All pages must be numbered consecutively and marked with your *exam number*. You should *not* write your name on the exam paper.
- *Your paper must be uploaded on the course page in Absalon at the given time.* The exam paper (including supporting material) must be in *PDF-format* and collected in *one file only*; the uploaded file must be named `1234.pdf`, where `1234` is your exam number.

Regarding the data for the exam paper, please note the following:

- All assignments are based on *different* data sets. You should use the data set located in the Excel file `Data1234.xls`, where `1234` is your exam number.
- To avoid that some data sets are more difficult to handle than others, the data sets are artificial (simulated from a known data generating process), and they behave, as close as possible, like actual data.

# 1 Background

This project examination considers models for consumption, income and unemployment in a small open economy based on annual data covering more than a century. The period covered, $1907 - 2016$, includes observations for the second world war, $1940 - 1945$, and the period around 1980 was characterized by marked changes in the rules for the international flow of goods and capital. Commentators have argued that these events may have fundamentally changed the functioning of the economy.

We consider a data set with $p = 5$ variables, in particular

$$x_t = (c_t : y_t : w_t : R_t : \Delta p_t : U_t)',$$

where the variables denote, respectively,

| | | |
|---|---|---|
| $c_t$ | : | log of the real private aggregate consumption |
| $y_t$ | : | log of real aggregate income |
| $w_t$ | : | log of real private wealth |
| $R_t$ | : | year-on-year interest rate on bonds |
| $\Delta p_t$ | : | year-on-year inflation rate |
| $U_t$ | : | Unemployment rate as a fraction of the labour force. |

Standard economic theory for consumption suggests that consumption in the long run depends on income and wealth, e.g. as a linear relationship,

$$c_t = \phi_1 y_t + \phi_2 w_t + u_{1t}, \tag{1.1}$$

where $u_{1t}$ measures the short-term deviation of consumption from its equilibrium value. In this case it may hold that $\phi_1 + \phi_2 = 1$. If the variables have unit roots and if we interpret the equilibrium as a cointegrating relationship, the steady-state in (1.1) would imply that $u_{1t}$ is a stationary process. An alternative version would state that consumption should be proportional to income in equilibrium, i.e.

$$c_t = \phi_3 y_t + u_{2t}, \tag{1.2}$$

where $u_{2t}$ is stationary and $\phi_3$ is likely to be unity. Similarly, private wealth may be proportional to income in the long-run as in

$$w_t = \phi_4 y_t + u_{3t}, \tag{1.3}$$

with $u_{3t}$ stationary. It addition, it could be the case that consumption, in either (1.1) or (1.2), depends on the *ex post* real interest rate,

$$R_t - \Delta p_t,$$

to capture the savings-incentive from the interest rate, or depends on the unemployment rate, $U_t$, to capture the consumption effect from the increased risk of income-loss implied by a high unemployment rate.

A different strand of theories suggests that the interest rate cointegrates with inflation, such that

$$R_t = \phi_5 \Delta p_t + u_{4t}, \tag{1.4}$$

with $u_{4t}$ stationary and where $\phi_5 = 1$ would correspond to the real interest rate. If the real interest rate is not stationary, it may cointegrate with income in an IS curve

$$y_t = \phi_6 t + \phi_7 (R_t - \Delta p_t) + u_{5t}, \tag{1.5}$$

with $u_{5t}$ stationary and where the trend captures the development in potential output or productivity. A final suggestion would be a link from production to the labour market, e.g. through a relationship of the form

$$U_t = \phi_8 (y_t - \phi_9 t) + u_{6t}, \tag{1.6}$$

with $u_{6t}$ stationary.

In the cointegration analysis below, we will take (1.1), (1.2), (1.3), (1.4), (1.5), and (1.6) as *theoretical* candidates for cointegrating relationships.

# 2 The Statistical Model

Let $x_t \in R^p$ and consider the vector autoregressive model:

$$x_t = \Pi_1 x_{t-1} + \Pi_2 x_{t-2} + ... + \Pi_k x_{t-k} + \phi D_t + \epsilon_t, \quad t = 1, 2, ..., T, \tag{2.1}$$

where $\epsilon_t$ is assumed to be independently Gaussian distributed, $N(0, \Omega)$, the initial values, $x_{-k+1}, ..., x_{-1}, x_0$, are considered fixed for the statistical analysis and the vector $D_t$ contains potential deterministic variables, such as a constant, a trend, and dummy variables relevant for the empirical analysis.

[1] Perform a graphical analysis by considering the given time series and relevant linear combinations, and give a brief description of the economic development in the country over the last century.

[2] Set up and estimate a model as (2.1) for the data in $x_t$. Explain the steps you take and motivate the choices you make in the process. In particular, you should motivate your choice of deterministic variables and discuss the potential presence of shifts in the equilibrium means.

State the assumptions for the model, and test that the model fulfills the assumptions. In practice it may not be possible to find a model that is acceptable in all directions, just do as well as you can.

[3] State the stability condition for the VAR model in (2.1), i.e. the condition under which $x_t$ generated from the equations in (2.1) are stationary processes.
Check the stability condition for the empirical model and comment on the results.

# 3 The Cointegration Rank

[4] Find the characteristic polynomial, $A(L)$, for your preferred version of the model in (2.1).

[5] Rewrite your preferred VAR model to error-correction form.
Show how $A(1)$ enters as a parameter in the error-correction form and explain the implication of unit roots on the parameters.

[6] Explain how to construct the likelihood ratio test for the cointegration rank, and discuss how the deterministic terms should be treated to obtain *similarity* of the test.

[7] Explain how the asymptotic distribution of the likelihood ratio statistic involving Brownian motions can be simulated using random walks and how the presence of a level shifts in the data changes the simulation of the limiting distribution.

[8] Determine the cointegration rank, $r$, for the data using all the available information.

# 4 Testing Hypotheses

[9] Impose the reduced rank, $\Pi = \alpha\beta'$ say, and estimate the cointegrated VAR model.
If your model includes deterministic variables restricted to the cointegration space, test if their coefficients are statistically significant or if they can be excluded from all cointegrating relations.
Also test if any of the endogenous variables can be excluded from all cointegrating relationships.
Explain what an *excludable variable* implies for the Granger representation, i.e. the moving average solution to the cointegrated VAR equations.

[10] Test if any of the variables in $x_t$ are stationary around the included deterministic variables.
Also test if they are stationary without allowing for deterministic variables.
Explain what a stationary variable implies for the Granger representation.

[11] Explain what the findings in questions [9] and [10] imply for the empirical relevance of the theoretical candidates for the cointegrating relations in (1.1), (1.2), (1.3), (1.4), (1.5), and (1.6).

[12] Explain how to test the hypothesis that a shock to certain variables, e.g. the shock $\epsilon_t^c$ to the variable $c_t$, has only transitory effects in the system, and test the hypothesis for

all variables in $x_t$.

# 5 Identification

[13] Based on the results from the hypotheses testing in Section 4, and based on the theoretical candidates, identify the cointegrating relationships in the empirical model. Give an economic interpretation of the long-run structure and the equilibrium adjustment.

[14] Calculate the parameters of the Granger representation and interpret the pushing forces in the model.

# 6 Robustness and Constancy

[15] Perform a recursive estimation and comment on the constancy of the parameters in the model.

[16] Explain the effects on the variables in $x_t$ of all the deterministic variables included in $D_t$ in (2.1).

[17] Remove all dummy and intervention variables from the model.
Reestimate your favorite identified model and comment on the importance of the dummy variables for the findings.

[18] Now consider the reduced sample $1955 - 2016$.
Reestimate your favorite identified model and comment on the importance of the length of the sample.

# 7 Extensions

[19] (Extending the Information Set) Consider now an extended data set by including also the replacement rate, i.e. the unemployment benefit as a ratio of wage income, called $Z_t$. We are informed that the replacement rate contains exactly the same stochastic trend as the unemployment rate, an no other stochastic trends.
Explain how your results of the cointegration analysis above would change if you considered the augmented data vector

$$x_t = (c_t : y_t : w_t : R_t : \Delta p_t : U_t : Z_t)'.$$

Explain in particular what would happen to the cointegration rank and the structure of the cointegrating relationships.

[20] (I(2) MODELLING) Imagine that we instead of $x_t = (c_t : y_t : w_t : R_t : \Delta p_t : U_t)'$ considered a set of nominal variables

$$Y_t = (c_t^n : y_t^n : w_t^n : R_t : p_t : U_t)',$$

where $p_t$ is the log of the price deflator, $c_t^n = c_t + p_t$ is the nominal consumption, $y_t^n = y_t + p_t$ is the nominal income, and $w_t^n = w_t + p_t$ is the nominal wealth. Assume that we have detected the presence of I(2) trends in the vector $Y_t$ and that we have estimated the I(2) cointegration model

$$\Delta^2 Y_t = \Pi Y_{t-1} - \Gamma \Delta Y_{t-1} + \sum_{i=1}^{k-2} \Psi_i \Delta^2 Y_{t-i} + \mu_0 + \mu_1 t + \epsilon_t,$$

subject to the two reduced rank restrictions

$$\begin{aligned} \Pi &= \alpha\beta' \\ \alpha_\perp' \Gamma \beta_\perp &= \xi\eta', \end{aligned}$$

where $\alpha, \beta \in \mathbb{R}^{p\times r}$, $\xi, \eta \in \mathbb{R}^{(p-r)\times s_1}$, and $s_2 = p - r - s_1$, where $r < p$ and $s_1 < p - r$.

[20.1] Assume first, that $r = 2$, $s_1 = 3$, and $s_2 = 1$ implying that there is a single I(2) stochastic trend, and assume that $\beta_{\perp 2} = \beta_\perp \eta_\perp$ is given by

$$\beta_{\perp 2} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}. \tag{7.1}$$

Explain how the common stochastic I(2) trend affects the variables.

Suggest a relevant *nominal-to-real transformation* such that the cointegration analysis can be performed within the I(1) cointegrated VAR model, and such that the polynomially cointegrating relationships can be recovered.

Explain how the restrictions implied by (7.1) can be tested within the I(2) cointegration model.

Also explain why the restrictions cannot be tested in an I(1) cointegrating analysis for $Y_t$.

[20.2] Next assume that $r = 2$, $s_1 = 2$, and $s_2 = 2$ implying that there are now two I(2) stochastic trends, and assume that

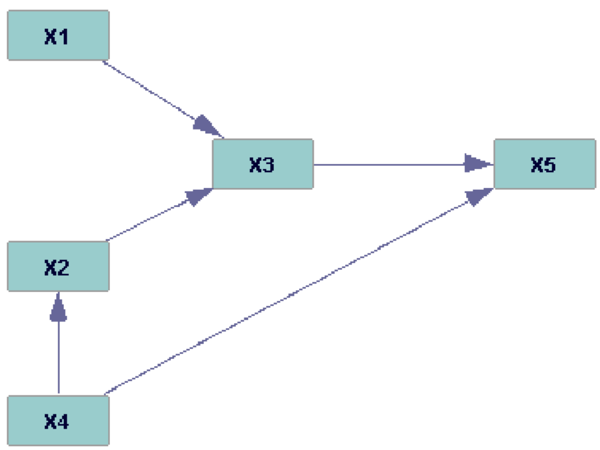
$$\beta_{\perp 2} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}. \tag{7.2}$$

Explain how the stochastic I(2) trends affect the variables.

Suggest a relevant *nominal-to-real transformation* such that the cointegration analysis can be performed within the I(1) cointegrated VAR model, and such that the polynomially cointegrating relationships can be recovered.

Explain how the restrictions implied by (7.2) can be tested within the I(2) cointegration model.

[21] (INFERENCE ON CONTEMPORANEOUS CAUSAL STRUCTURES) Consider the five-dimensional system of $X = (X_1 : X_2 : X_3 : X_4 : X_5)'$ and a true causal structure as given by




(7.3)

To recover the class of observationally equivalent causal structures we begin with the fully saturated skeleton of a graph, where all variables in $X$ are linked by undirected edges.

Explain which independence and conditional independence restrictions that are implied by the true causal structure in (7.3) and explain how they can be used in a causal search algorithm to construct a simplified but undirected skeleton of the graph.

Explain which information you need in order to orient the edges in the undirected graph. Next state the conditional dependence structures you can derive from the true causal structure, and use those to orient as many edges as you can.

How many members does the class of observationally equivalent structures contain in this case?